

Math 300-0 Final project - Groups

Groups are mathematical objects that describe symmetry. You've already worked groups like $\mathbb{Z}$ and $\mathbb{Q}$, but there are many more types of groups. This project will introduce a few intuitive groups and group axioms, along with some group properties.

Required materials

The reference for this project is *Contemporary Abstract Algebra* by Joseph A. Gallian. A PDF of the relevant pages is available in for free on Canvas.

Day 1: Dihedral groups

To do before May 20

- Read Gallian, Chapter 1, pp27-33.
 - What is a dihedral group? What do the elements of dihedral group D_4 represent, geometrically?
 - What does it mean for a group to be abelian? Can you think of objects you know whose multiplication is non-abelian?
- Write any remaining questions you have in the shared Overleaf document.

To do after May 20

Write solutions to the following exercises.

Problem 1. Exercise 11

Problem 2. Exercise 13

Problem 3. Exercise 15

Day 2: Groups

To do before May 24

- Read Gallian chapter 2, pp 39-52. You may skip examples about matrices (6, 9, 10, 18, 19) and complex numbers (15, 16). You may skip the Historical Note. Note that Example 7 and onward, this text uses the notation $\mathbb{Z}_n$ to mean “integers modulo n ”, whereas we used $\mathbb{Z}/n\mathbb{Z}$. Also, the elements of $\mathbb{Z}_n$ are denoted without brackets in this book.
- Think about these questions. If you are not able to answer them, consider reviewing the text again.
 - What is the identity element of $\mathbb{Z}$ under addition? What is the identity element of $\mathbb{Z}$ under multiplication?
 - What is an example of a set with an operation that is not a group?
- Write any remaining questions you have in the shared Overleaf document.

To do after May 24

Write solutions to the following exercises.

Problem 4. Exercise 5a

Problem 5. Exercise 15

Problem 6. Exercise 20

Day 3: Subgroups

To do before May 28

- Read Gallian chapter 3 pp59-66, up to and including Example 14.
- Think about these questions. If you are not able to answer them, consider reviewing the text again.
 - What is the order of the class $[1]$ in $\mathbb{Z}_5$?
 - What does it mean for an element to have infinite order?
 - Is D_n always a subgroup of D_{2n} ?
- Write any remaining questions you have in the shared Overleaf document.

To do after May 28

Write solutions to the following exercises.

Problem 7. Exercise 1, only for $\mathbb{Z}_{12}$ and D_4

Problem 8. Exercise 4

Final Project submission

Submit your typed exercises on Canvas (not Crowdmark!) by **5pm on June 3**. Remember, collaboration and using outside resources is encouraged, but your write-up must be your own work.